DE/Theory I.
The leading term in the Lagrangian at low energies and long distances is \( \Lambda \)

\[
\mathcal{S} = \int d^4x \sqrt{-\mathcal{G}} \, M_p^2 (-\Lambda + \mathcal{R} + \cdots) 
\]

can be constant c.c. or potential energy \( V(\phi) = -\Lambda M^2 \) for scalar \( \phi \).

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{M_p^2} \left( \Lambda M_p^2 - \frac{k}{a^2} + \frac{l_m}{a^3} + \cdots \right) 
\]

with \( ds^2 = -dt^2 + a^2(t) \, ds_{\text{space}}^2 \)
Quantum effects generate $\Lambda$ (cf scalar masses); a free \{Boson, Fermion\} contributes

$$\Delta \Lambda = \pm \int d^3k \frac{1}{2} \omega_k - \pm M_{\text{cutoff}}^4$$

\[\text{o-point energy}\]

\[\uparrow \text{sensitive to high-scale physics}\]

Unbroken SUSY would cancel this; dynamically broken SUSY would

$\rightarrow M_{\text{cutoff}} \sim M_{\text{susy}} \sim e^{-\frac{\text{const}}{g^2}}$

but particle physics is non-SUSY up to at least $M_{\text{susy}} \geq \text{TeV}$.
At this level, the mystery of DE is why it is so small, not why it is there.

This may be a selection effect: structure does not form in a (part of the) universe with

\( \Lambda \gg \) its observed value

\( \sim 10^{-120} M_p^2 \)

Weinberg; Linde

\( \rightarrow \) Expect to see \( \Lambda \) as big as can be consistent with structure.
This prediction required that different values of $\Lambda$ are dynamically accessible (including the small, fine-tuned values).

\[ \Lambda \]

Weinberg assumed $\sim$ uniform distribution near $\Lambda = 0$.

Such a scenario has roots in inflationary cosmology...

Guth Linde Albrecht/Steinhardt
... and more recently came out in string theory:

→ Rich scalar potential

\[ U(\Phi_i, \Theta_i) \]

\( i \) size + shape of extra dimensions

\( i \) angular scalars: "axions" + duals.
Rich Potential Landscape

\[ U(\Phi_i, \Theta_j) \]

\[ \frac{4 \Phi}{M_p^4} \]

\[ e^{4\Phi} \times \left\{ (D-10)e^{-2\Phi} + \int \text{Re} e^{-2\Phi} \right\} \]

- Tension of "orientifold defects"

\[ \int dC_p + B dC_{p-2} \]

- Tension of "branes"

\[ \cdots + \text{quantum effects} \]
Many competing forces suggest "discretuum" of metastable values of $\Lambda$.

E.g. fluxes: \[ F = dA \]

\[
\exp\left( i \oint F \right) = \exp\left( i \oint A \right)
\]

\[
\gamma = e^{i \oint A}
\]

\[ \text{charged particle} \]

\[ \int F = \text{integer} \]

Stokes
\[
\int F = F \times \left( \frac{R^2}{\text{area of } S^2} \right) = \mathcal{N} \Rightarrow U_{\text{flux}} \propto \frac{\mathcal{N}^2}{R^4}
\]

- **Multiple fluxes**

\[
U_{\text{flux}} \propto \sum_{i=1}^{\# \text{ fluxes}} \frac{N_i^2}{R_i^4}
\]

Bousso-Polchinski toy model

\[
\Lambda = -|\mathcal{N}_0| + \sum n_i^2 q_i^2
\]

Figure 1: The allowed values of the four-form energy density are given by the radius-squared of points in the grid, whose dimension is the number of four-forms \(J\). The spacing in direction \(i\) is \(q_i\). The negative of the bare cosmological constant corresponds to a \((J - 1)\)-dimensional sphere, and cancellation is possible if there is at least one grid point sufficiently close to the sphere.
More complete treatment of stabilization of meta-stable $\Lambda > 0$ "vacua".

$\Rightarrow$ detailed mechanisms for $\Lambda = 0$ (tunneling decay to MSS, GKP, KKLT, ...)

$g, \frac{l_s}{L_1}, \ldots, \frac{l_s}{L_n}$

e.g. KKLT

$g_{00} \ll 1$

Calabi-Yau (accomodates low-energy SUSY)

No explicit calculation of tuned models $\Lambda = 10^{-120} M_p^2$

- Lines up with Weinberg's argument.
- Conceptual questions: how to formulate physics in cosmological backgrounds with horizons and singularities?
Another possibility, somewhat similar to inflation, is DE due to a rolling scalar field.

\[ S_\phi = \int d^4x \sqrt{-g} L(\phi, (\partial \phi)^2) \]

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 \approx \frac{V}{M_p^2} \gg \frac{\rho_{\text{kinetic}}}{M_p^2} \]

\[ \frac{\dot{H}}{H^2} < 1, \quad \frac{\ddot{H}}{H^2} < 1 \]

\[ \rightarrow \text{accelerated expansion } \ddot{a}(t) > 0 \]
e.g. **Slow-roll Quintessence:**

\[ H^2M_p^2 = 10^{-120} \quad M_p^{-4} \sim (10^{-3} \text{ eV})^4 \]

\[ \frac{V'}{V} M_p \leq 1, \quad \frac{V''}{V} M_p^2 \leq 1 \]

*For* \( V(\phi) = M^4 \cos \frac{\phi}{f} \)

or *for* \( V(\phi) = M^{4-p} \phi^p \)

\[ \Rightarrow \phi \geq M_p \]

Similar to chaotic inflation

or "natural inflation"

\[ V \sim (10^{-3} \text{ eV})^4 \Rightarrow \left( \frac{\mu}{M_p} \right)^{4-p} \left( \frac{10^{-3} \text{ eV}}{M_p} \right)^4 \]

\[ \mu^{4-p} M_p \]
e.g. for $V = m^2 \phi^2$

one needs $\phi \gtrsim M_p$

$m \sim H_0 \sim 10^{-60} M_p$

So $m^2 \sim \frac{\text{TeV}^{16}}{M_p^{12}}$

=) even optimistically assuming $M_{\text{Susy}} = \text{TeV}$, this is sensitive to dimension-16 Planck-suppressed operators in the effective theory.
Both Inflation and DE (c.c. or quintessence) are "UV-Sensitive": depend on assumptions about high-scale physics.

1. $\frac{\dot{H}}{H^2}, \frac{\ddot{H}}{H^3}$ get $O(1)$ corrections from e.g. $\Delta L = \frac{V(\alpha-\alpha_0)^2}{M_p^2}$ even though $M_p$ suppressed.

2. For models with a large field range $\Delta \Phi \geq M_p$, sensitive to an sequence of such operators, or to $F$ of approx. shift-symmetry $\Phi \to \Phi + c$

3. Quintessence: requires symmetry to avoid generating time-varying $\phi_{\text{AED}}$. 
Axions naturally respect an (approximate) shift symmetry $\phi \rightarrow \phi + C$ and (couple via their derivatives).

Pseudoscalars don't induce $\phi_{\text{QED}}(t)$.

$$S_Q = \int d^4x \frac{\mathcal{L}}{f} \frac{F}{\sqrt{\Theta}} \frac{F}{\sqrt{\Theta}} + \Delta S_{Q}$$

"Natural Inflation"/Quintessence

$\Theta \rightarrow \Theta + (2\pi)^2$

$\phi_\alpha = f_\alpha \Theta$

---

Freese, Frieman, Olinto '90; +Adams, BOND '93
→ Does $\frac{\Delta Q}{M_p} \geq 1$, protected by shift symmetry, arise in string theory?

For axions, $f << M_p$

in currently controlled regions of the landscape. (size $L \gg M_p^{-1}$)

$$\int d^4x \sqrt{-g} |d C |^2 = \int d^4x \sqrt{-g} \frac{M_p^2}{(L M_p)^{2p}} (\partial \Theta)^2$$

$g_{\alpha \beta} \ldots g^{(\alpha_1 \beta_1)}$ pt pt

* BTW, not "anything goes" in the landscape!
But must take into account "monodromy" in string compactifications

unwraps the would-be periodic direction. $\rightarrow$ Large field range with distinctive potential

$$V(\phi > M_p) \sim \begin{cases} \alpha^{2/3} & \text{twisted torus} \\ \alpha, \alpha^{1/5}, \ldots & \text{axions} \end{cases}$$
The basic mechanism is very simple:

- "NS5" branes position periodic on this circle, until add stretched "D4" brane predicts flattened potential for inflation/quintessence:
  - Log-distributed
  - Slopes so far... but fluxes may help.

["T-dual" to axions]
Primordial inflation:
- Solves macroscopic puzzles
- Must exit to radiation, matter domination ⇒ require scalar field φ
- φ's quantum fluctuations seed structure super-horizon perturbations

→ details of <φ(x₁)⋯φ(xₙ)>, and gravity waves, can observationally distinguish broad classes of mechanisms.
- Unknown scale, but can be high, ~M⁴
- Couplings to Standard model
  → (re-)heating
Quintessence (rolling scalar DE)

- Does not solve c.c. problem; extra scalar \( \Rightarrow \) extra small number to explain (can be obtained dynamically).

- Scale known, \( \mu << \) most other scales in physics

- Consistent with Linde/Weinberg argument if scan slopes uniformly.
The quest for a viable "Wilsonian-natural" explanation for the hierarchy $\frac{\Lambda}{M_p^2} \sim 10^{-120}$ continues, without success yet.

(Might hope that heavy particles/strings conspire to cancel $\Delta \Lambda$, since all contribute. Might hope that modifying GR could help, but these introduce instabilities, are difficult to UV-complete, or turn out equivalent to adding a scalar.)

Selection + Multiverse is so far the most conservative option (!).
Mysteries of DE

$w < -\frac{1}{3} \Rightarrow \text{horizon}$

e.g. dS $w = -1$

Including decays:

Dynamical connections between different $\Lambda$, $\phi$, different D, topology, fluxes, ...
$\Lambda < 0$ (AdS$_4$) has a precise non-perturbative formulation. Maldacena

4d GR + strings $\rightarrow$ QFT$_3$, no gravity

$\mathcal{S} \cong \text{CFT}_3$

many distinct examples, don't mix, no horizon

Observables = QFT Correlation Functions.
For $\Lambda = 0$, the general framework (an $S$-matrix is also known)

An observer can causally collect the results.

In contrast, in $dS$:

Classically appears as though info lost (cf Black Holes)
Attempts to build up from AdS/CFT make some progress, but leave propagating Gravity in the dual

\[ ds/dS = \frac{\sin^2 \frac{w}{L}}{dS_4} ds_4^2 + dw^2 \]

\[ \Rightarrow 2 \text{ CFT}_3 + G R_3 \]

- UV-complete construction, concrete dual \( \rightarrow dS_4 \) entropy (order of magnitude)

\[ \text{FRW/CFT} \]

\[ \Rightarrow \text{Euclidean AdS}_3/\text{CFT}_2 \]

- Attempts at probability "measures" interesting but still ad hoc.
Summary

- DE Mysterious because
  - Small (but not mysterious if selection effect)
  - Requires new framework
- DE is UV-sensitive
  - Can parameterize in GR+QFT but makes assumptions about high-scale physics.
- Primordial DE (i.e. inflation) has addition experimental + theoretical handles. Tilt $n_s$ Gravity waves $r$ Non-Gaussianity $<S>$
- Ongoing attempts for "natural" explanation
- Various attempts at exotic alternatives