Applications of AdS/QCD and Light-Front Holography to Hadron Physics

A New Perspectives on QCD Condensates and Dark Energy

Experimental and Theoretical Challenges to Probing Dark Energy

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Stanford University
“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

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\[ (\Omega_\Lambda)_{QCD} \sim 10^{45} \]
\[ (\Omega_\Lambda)_{EW} \sim 10^{56} \]
\[ \Omega_\Lambda = 0.76 (\text{expt}) \]

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb
“Condensates in Quantum Chromodynamics and the Cosmological Constant.”
“Dark Energy”

Enormous and even greater contribution from Higgs VEV!

- **QCD vacuum contribution**

- If chiral symmetry breaking is expressed in a nonzero expectation value of the quark bilinear, then the energy difference between the symmetric and broken phases is of order

\[ M_{QCD} \approx 0.3 \text{ GeV} \]

- Mass-scale generated by spacetime-independent condensate

- One obtains therefrom:

\[ \rho_{\Lambda}^{QCD} = 10^{46} \rho_{\Lambda}^{obs} \]

“The biggest embarrassment in theoretical physics.”
Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind

Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.\(^1\) Because of an instability of the chirally invariant vacuum, the real vacuum is “aligned” into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.\(^2\) A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron’s wave function and not to the vacuum.\(^3\)
New perspectives on the quark condensate

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We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.
Gell-Mann - Oakes - Renner Relation (1968)

\[ \frac{f_\pi^2}{m_\pi^2} = -2 m(\zeta) \langle \bar{q}q \rangle^\zeta \]

- Pion’s leptonic decay constant, mass-dimensioned observable which describes rate of process \( \pi^+ \rightarrow \mu^+\nu \)
- Vacuum quark condensate

How is this expression modified and interpreted in a theory with confinement?
Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

\[ \langle B | \bar{q}q | B \rangle \neq \langle 0 | \bar{q}q | 0 \rangle \]
Bethe-Salpeter Analysis

\[ f_H P^\mu = Z_2 \int_\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} \left[ T_H \gamma_5 \gamma^\mu S \left( \frac{1}{2} P + q \right) \right] \Gamma_H (q; P) S \left( \frac{1}{2} P - q \right) \]

\( f_H \) Meson Decay Constant  
\( T_H \) flavor projection operator,  
\( Z_2(\Lambda), Z_4(\Lambda) \) renormalization constants  
\( S(p) \) dressed quark propagator  
\( \Gamma_H(q; P) = F.T. \langle H|\psi(x_a)\bar{\psi}(x_b)|0\rangle \) Bethe-Salpeter bound-state vertex amplitude.

\[ i \rho^H_\zeta \equiv -\frac{\langle q\bar{q} \rangle^H_\zeta}{f_H} = Z_4 \int_\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} \left[ T_H \gamma_5 S \left( \frac{1}{2} P + q \right) \right] \Gamma_H (q; P) S \left( \frac{1}{2} P - q \right) \]

\( i \rho^\pi \equiv -\frac{\langle q\bar{q} \rangle^\pi}{f_H} = Z_4 \int_\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} \left[ T_H \gamma_5 S \left( \frac{1}{2} P + q \right) \right] \Gamma_H (q; P) S \left( \frac{1}{2} P - q \right) \]

**In-Hadron Condensate!**  
\[ i \rho^\pi = < 0|\bar{q}\gamma_5 q|\pi > \]

\[ f_H m_H^2 = -\rho^H_\zeta M_H \quad M_H = \sum_{q \in H} m_q \]

\[ m_\pi^2 \propto (m_q + m_{\bar{q}}) / f_\pi \quad \text{G-MOR} \]
Pseudoscalar projection of pion’s Bethe-Salpeter wavefunction onto the origin in configuration space
– or the pseudoscalar pion-to-vacuum matrix element

\[
i \rho_{\pi} = -\langle 0 | i \gamma_5 q | \pi \rangle
\]

\[
= Z_4(\zeta, \Lambda) \, \text{tr}_{CD} \int_0^\Lambda \frac{d^4 q}{(2\pi)^4} \gamma_5 S(q^+) \Gamma_\pi(q; P) S(q^-)
\]

Rigorously defined in QCD – gauge-independent, cutoff-independent, etc.

- For arbitrary current-quark masses
- For any pseudoscalar meson
e-Print: nucl-th/9707003

Pi- and K meson Bethe-Salpeter amplitudes.
e-Print: nucl-th/9708029

Concerning the quark condensate.
e-Print: nucl-th/0301024

“\textit{In-Meson Condensate}”

\[-\langle \bar{q} q \rangle_{\xi}^\pi = f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle.\]

Valid even for $m_q \to 0$$f_\pi$ nonzero

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QCD Condensates

Stan Brodsky, SLAC
**Higher Light-Front Fock State of Pion Simulates DCSB**

\[ f_\pi P^+ = \langle 0 | \bar{q} \gamma^5 \gamma^+ q | \pi \rangle \]

**Instantaneous quark propagator contribution to \( \rho_\pi \) derived from higher Fock state**

\[ i \rho_\pi = \langle 0 | \bar{q} \gamma^5 q | \pi \rangle \]

**Higher Fock state acts like mass insertion**

Roberts, Tandy, Shrock, sjb
Determinations of the vacuum Gluon Condensate

\[ \langle 0| \frac{\alpha_s}{\pi} G^2 |0 \rangle \text{ [GeV}^4] \]

-0.005 ± 0.003 from \( \tau \) decay. Davier et al.

+0.006 ± 0.012 from \( \tau \) decay. Geshkenbein, Ioffe, Zyablyuk

+0.009 ± 0.007 from charmonium sum rules Ioffe, Zyablyuk

Consistent with zero vacuum condensate
Paradigm shift: In-Hadron Condensates

**Resolution**

- Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, “condensates” do not exist as spacetime-independent mass-scales that fill all spacetime.

- *So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

- No qualitative difference between $f_\pi$ and $\rho_\pi$

**Chiral limit**

$$\kappa_\pi(0; \zeta) = -<\bar{q}q>^0_\zeta$$
Quark and Gluon condensates reside within hadrons, not vacuum

- Bound-State Dyson Schwinger Equations
- AdS/QCD
- Analogous to finite size superconductor
- Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict

R. Shrock, sjb
ArXiv:0905.1151
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“Condensates in Quantum Chromodynamics and the Cosmological Constant.”
Dirac’s Amazing Idea:
The Front Form

Evolve in ordinary time

\[ \sigma = ct - z \]

Evolve in light-front time!

\[ \tau = t + z/c \]

Instant Form

Front Form

Plessas: Point Form
Each element of flash photograph illuminated at same Light Front time

\[ \tau = t + \frac{z}{c} \]

Evolve in LF time

\[ P^- = i \frac{d}{d\tau} \]

Causal, Trivial Vacuum

zero !!
General remarks about orbital angular momentum

\[ \mathbf{\vec{R}} \perp \mathbf{x} \mathbf{\vec{R}} \perp + \mathbf{\vec{b}} \perp \mathbf{i} \]

\[ \sum_{i} n_{i} \mathbf{\vec{b}} \perp i = 0 \]

\[ \sum_{i} n_{i} \mathbf{x} i = 1 \]

\[ \sum_{i} n_{i} \mathbf{k} \perp i = \mathbf{\vec{0}} \perp \]

\[ x = \frac{k^{+}}{P^{+}} = \frac{k^{0} + k^{3}}{P^{0} + P^{3}} \]

**Fixed \( \tau = t + z/c \)**

**Process Independent**
**Direct Link to QCD Lagrangian!**

\[ \Psi_{n}(x_{i}, \mathbf{\vec{k}}_{\perp i}, \lambda_{i}) \]

**Invariant under boosts! Independent of \( P^{\mu} \)**

**Plus momenta conserved; all \( k^{+} \geq 0 \)**

**Causal, Trivial Vacuum.**

*Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory*
Wick Theorem

Feynman diagram = sum n!
instant-form time-ordered diagrams

\[ a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} \]
Wick Theorem

Feynman diagram = one front-form time-ordered diagram!

Also \( P \to \infty \) observer frame (Weinberg)

\[
a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}
\]
Calculation of Form Factors in Equal-Time Theory

**Instant Form**

\[
\sum \quad = q^+ + q^+ + \text{zero}!!
\]

Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

**Front Form**

\[
\sum \quad = q^+ + q^+ + \text{zero}!!
\]

Complete Answer

Absent for \( q^+ = 0 \)

No vacuum graphs

Peking University
October 27, 2010

Applications of Light-Front Holography

Stan Brodsky
SLAC
\[
\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{1}{2} \times 
\]

\[
[- \frac{1}{qL} \psi_a^\dagger(x_i, k'_\perp i, \lambda_i) \psi_a(x_i, k_\perp i, \lambda_i) + \frac{1}{qR} \psi_a^\dagger(x_i, k'_\perp i, \lambda_i) \psi_a(x_i, k_\perp i, \lambda_i)]
\]

\[
k'_{\perp i} = k_\perp i - x_i q_\perp \quad \quad \quad \quad k'_{\perp j} = k_\perp j + (1 - x_j) q_\perp
\]

\[
q_{R,L} = q^x \pm iq^y
\]

Must have \(\Delta \ell_z = \pm 1\) to have nonzero \(F_2(q^2)\)

Nonzero Proton Anomalous Moment \(\Rightarrow\)
Nonzero orbital quark angular momentum

Peking University
October 27, 2010

Applications of Light-Front Holography

Stan Brodsky
SLAC
Anomalous gravitomagnetic moment $B(0)$

Terayev, Okun, et al: $B(0)$ must vanish because of Equivalence Theorem

Hwang, Schmidt, sjb;
Holstein et al

$B(0) = 0$

Each Fock State
Light Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements.

\[ \psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta) \]

\[ \zeta = \sqrt{x(1-x)b_{\perp}^2} \]

\[ \psi(x, b_{\perp}) \leftrightarrow \phi(z) \]

\[ LF(3+1) \leftrightarrow AdS_5 \]

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